



The Ohio State University

ENGINEERING CALCULATIONS FOR  
COMMUNICATIONS SATELLITE SYSTEMS PLANNING

C.H. Reilly  
C.A. Levis  
C. Mount-Campbell  
D.J. Gonsalvez  
C.W. Wang  
Y. Yamamura



The Ohio State University

**ElectroScience Laboratory**

Department of Electrical Engineering  
Columbus, Ohio 43212

Interim Report 716548-2  
Grant No. NAG 3-159  
7/15/84 - 7/14/85

(NASA-CR-176555) ENGINEERING CALCULATIONS  
FOR COMMUNICATIONS SATELLITE SYSTEMS  
PLANNING Interim Report, 16 Jul. 1984 - 15  
Jul. 1985 (Ohio State Univ.) 46 p  
C A03/MF A01

N86-19493

Unclas

CSSL 17B G3/32 05475

National Aeronautics and Space Administration  
Lewis Research Center  
21000 Brookpark Road  
Cleveland, Ohio 44135

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<b>REPORT DOCUMENTATION PAGE</b>		<b>1. REPORT NO.</b> 716548-2	<b>2.</b>	<b>3. Recipient's Accession No</b>
<b>4. Title and Subtitle</b> ENGINEERING CALCULATIONS FOR COMMUNICATIONS SATELLITE SYSTEMS PLANNING				<b>5. Report Date</b> 7/16/84 - 7/15/85
<b>7. Author(s)</b> C.H. Reilly, C.A. Levis, C. Mount-Campbell, D.J. Gonsalvez, C.W. Wang, Y. Yamamura				<b>8. Performing Organization Rept. No.</b> 716548-2
<b>9. Performing Organization Name and Address</b> The Ohio State University ElectroScience Laboratory Department of Electrical Engineering 1320 Kinnear Road Columbus, Ohio 43212				<b>10. Project/Task/Work Unit No.</b>
				<b>11. Contract(C) or Grant(G) No</b> (C) (G) NAG 3-159
<b>12. Sponsoring Organization Name and Address</b> National Aeronautica and Space Administration Lewis Research Center 21000 Brookpark Rd. Cleveland, Ohio 44135				<b>13. Type of Report &amp; Period Covered</b> Interim
<b>15. Supplementary Notes</b>				<b>14.</b>
<b>16. Abstract (Limit: 200 words)</b>  This report deals with computer-based techniques for optimizing communications-satellite orbit and frequency assignments. A gradient-search code was tested against a BSS scenario derived from the RARC-83 deliveries. Improvement was obtained, but each iteration requires about 50 minutes of IBM-3081 CPU time. Gradient-search experiments on a small FSS test problem, consisting of a single service area served by 8 satellites, showed quickest convergence when the satellites were all initially placed near the center of the available orbital arc with moderate spacing. When they were initially grouped near a boundary of the arc, initial improvement was small, but dramatic improvement was obtained later in some cases. This shows the importance of not terminating the gradient search process prematurely. A transformation technique is proposed for investigating the surface topography of the objective function used in the gradient-search method. A new synthesis approach is based on transforming single-entry interference constraints into corresponding constraints on satellite spacings. These constraints are used with linear objective functions to formulate the co-channel orbital assignment task as a linear-programming (LP) problem or mixed integer programming (MIP) problem. Globally optimal solutions are always found with the MIP problems, but not necessarily with the LP problems. The MIP solutions can be used to evaluate the quality of the LP solutions. The initial results are very encouraging.				
<b>17. Document Analysis a. Descriptors</b>  Satellite Broadcasting Service Fixed Service Orbit  Frequency Assignment Regulation Interference				
<b>b. Identifiers/Open-Ended Terms</b>				
<b>c. COSATI Field/Group</b>				
<b>18. Availability Statement</b>		<b>19. Security Class (This Report)</b> Unclassified	<b>21. No. of Pages</b> 40	
		<b>20. Security Class (This Page)</b>	<b>22. Price</b>	

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## I. PURPOSE

The purpose of this grant is to develop methods and procedures, including computer codes, for performing engineering calculations which will be useful for the United States delegations to international administrative conferences concerning satellite communications. During the interim 15 July 1984 to 14 July 1985, attention has been directed toward both the Broadcasting Satellite Service (BSS) and the Fixed Satellite Service (FSS). However, most of our effort was devoted to FSS issues since this service will be a topic at the World Administrative Radio Conferences in 1985 (WARC-85) and 1988 (WARC-88).

## II. IMPLEMENTATION OF THE BSS CODE FOR THE RARC-83 SCENARIO

Despite significant effort dedicated to running the gradient search code developed for BSS synthesis for the RARC-83 scenario [1,2], a complete run has not yet been made. The problem has been to obtain or create a set of compatible input data files which have real relevance to the RARC-83 agreement. Before studying this scenario in detail, we could not fully appreciate its intricacies. Two aspects which our code was not prepared to handle are the decision to ignore certain interferences on an ad-hoc basis, as specified by an interference matrix, and the decision to serve several diverse administrations by means of a common satellite. Code modifications to allow these two options are now underway.

Meanwhile, a preliminary diagnostic run was made without these options. A single iteration of the gradient search algorithm took about 40 CPU minutes on the IBM 3081 computer. The next iteration was not completed in the next 60 CPU minutes. Although those computer times are less significant in light of the changes we anticipate making, they do point out that, when the gradient search algorithm is applied to a large problem, long solution times are probable.

### III. FSS/GRADIENT SEARCH EXPERIMENTS

#### A. USA-EAST TEST PROBLEM

A new test problem was used to experiment further with the gradient search algorithm. For this test problem, we considered a single service area being served by eight FSS satellites. Each of these satellites is assumed to have access to the entire available frequency spectrum, so that a co-channel calculation is sufficient. For convenience, the service area was defined by the USA-East test points of the RARC-83 scenario, but FSS antenna patterns were used.

#### B. COMPUTER RESULTS

About twenty computational experiments were conducted using this new test problem, with one change in the previously reported algorithm [1,2]. In the coarse mode, the objective function is still evaluated at ten equally spaced points lying in the negative gradient direction between the current solution and the feasible region boundary, but now



the nearest point at which improvement was found becomes the next solution in the iterative process. It would not be practical to present all of the results here; however, we do include those which we believe are the four most significant runs. The full set will be presented in a technical report, yet to be written.

Based on the results of the RSS gradient search experiments conducted earlier, we had surmised that a good solution to a synthesis test problem is often obtained rather quickly if the satellites are initially assumed to be collocated. For the test problem considered here, the satellites cannot be precisely collocated at the outset. If they are, all the components of the gradient are identical; hence the satellites move in exactly the same manner, and no separation between satellites can ever be achieved.

Some experiments were made in which the satellites were nearly collocated, separated by  $0.1^\circ$ . Figures 1 and 2 summarize the results (satellite locations and worst C/I ratio, respectively) by iteration for a run of 10 iterations (Run 1) in which the satellites were initially almost collocated near the center of the feasible orbital arc, which extended from  $62^\circ\text{W}$  to  $100^\circ\text{W}$ . We see that a fairly good solution is obtained after 10 iterations: the satellites are spread out almost uniformly over the feasible orbital arc, and the worst C/I ratio exceeds 27 dB.

For Run 2, the satellites were separated by  $2^\circ$  and centered over the feasible arc,  $62^\circ\text{W}$  to  $100^\circ\text{W}$ . The results for 10 iterations are displayed in Figures 3 and 4. A better solution is found with Run 2

than was found with Run 1. The worst C/I ratio is almost 30 dB. The satellites are more nearly uniformly spread out than they were at the end of Run 1.

The eight satellites were also initially separated by  $0.1^\circ$  for Run 3; however, they were positioned at the eastern boundary of the feasible arc, rather than being centered over the arc. Figures 5 and 6 present the results for Run 3. We see that there is very little movement of the satellites in the 10 iterations carried out. Not surprisingly, there is correspondingly little improvement in the C/I ratios; in fact, the worst C/I ratio for each satellite is still negative at the end of the run. It appears from this and other test problems that the rate of convergence to an acceptable solution is slow when all the satellites are positioned near a boundary of the feasible arc. We noticed a similar phenomenon earlier with the BSS test problem. Twenty additional iterations of the gradient search algorithm were performed with very little further improvement in the solution.

The separation between most pairs of satellites was increased to  $1^\circ$  for Run 4. Two of the satellites, satellites 4 and 5, were separated by only  $0.1^\circ$ . As in Run 3, all of the satellites were positioned initially near the eastern boundary of the feasible arc. Unlike the three runs described above, thirty iterations of the gradient search algorithm were executed for Run 4. The results by iteration for this run are shown in Figures 7a, 7b, and 7c (satellite locations) and 8a, 8b, and 8c (worst C/I ratios).

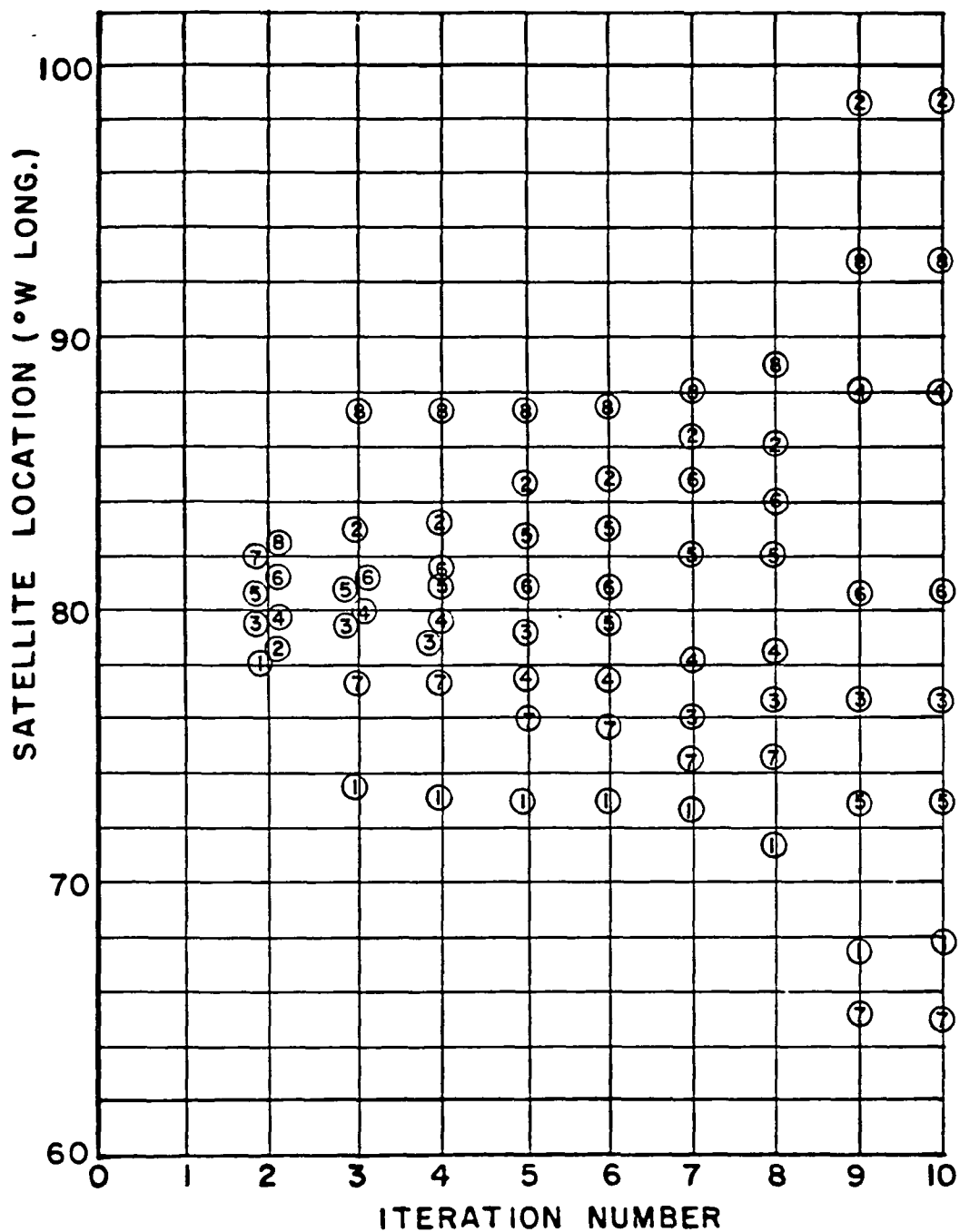


Figure 1. Satellite locations for Run 1 by iteration.

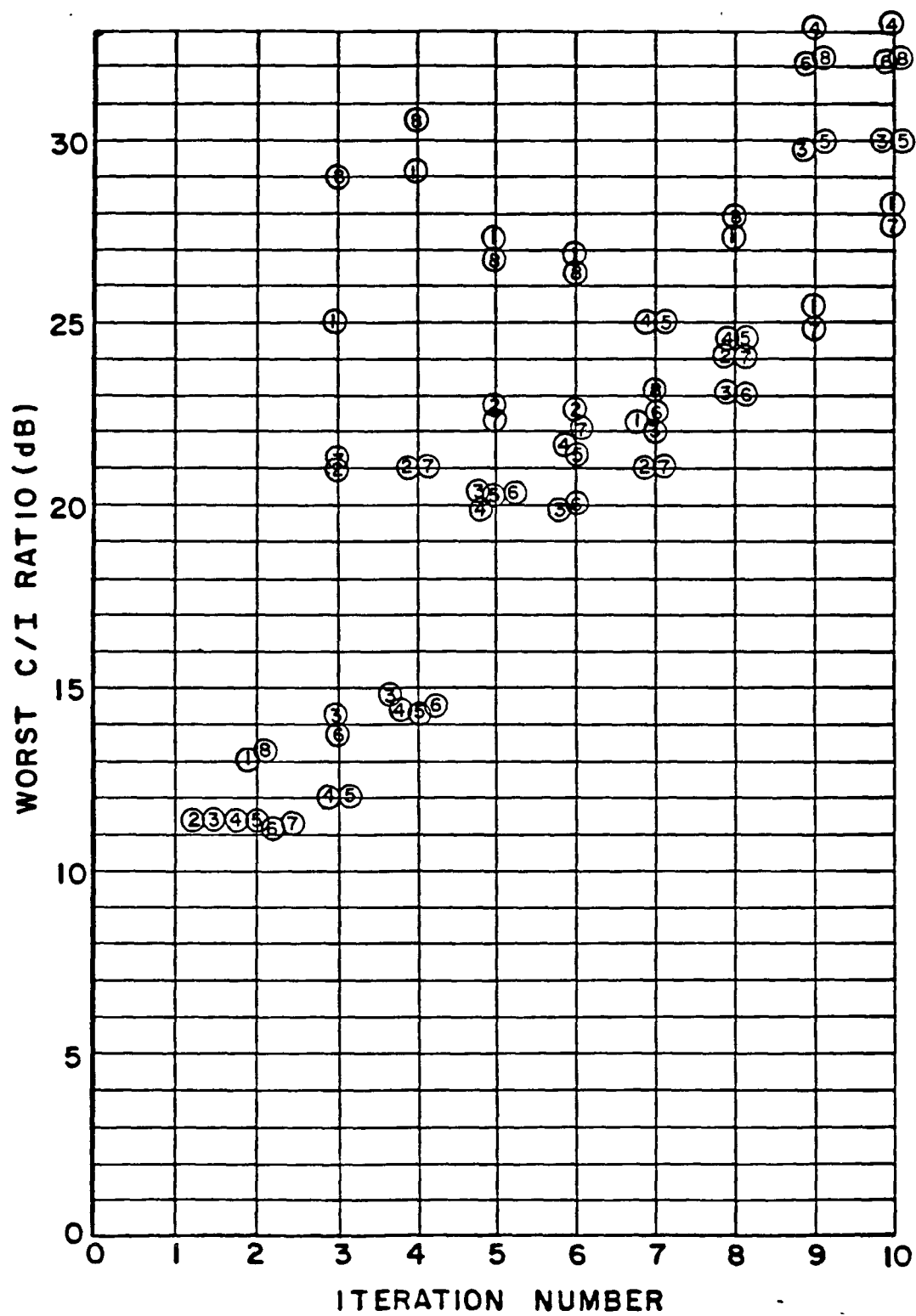


Figure 2. Worst C/I ratios for Run 1 by iteration.

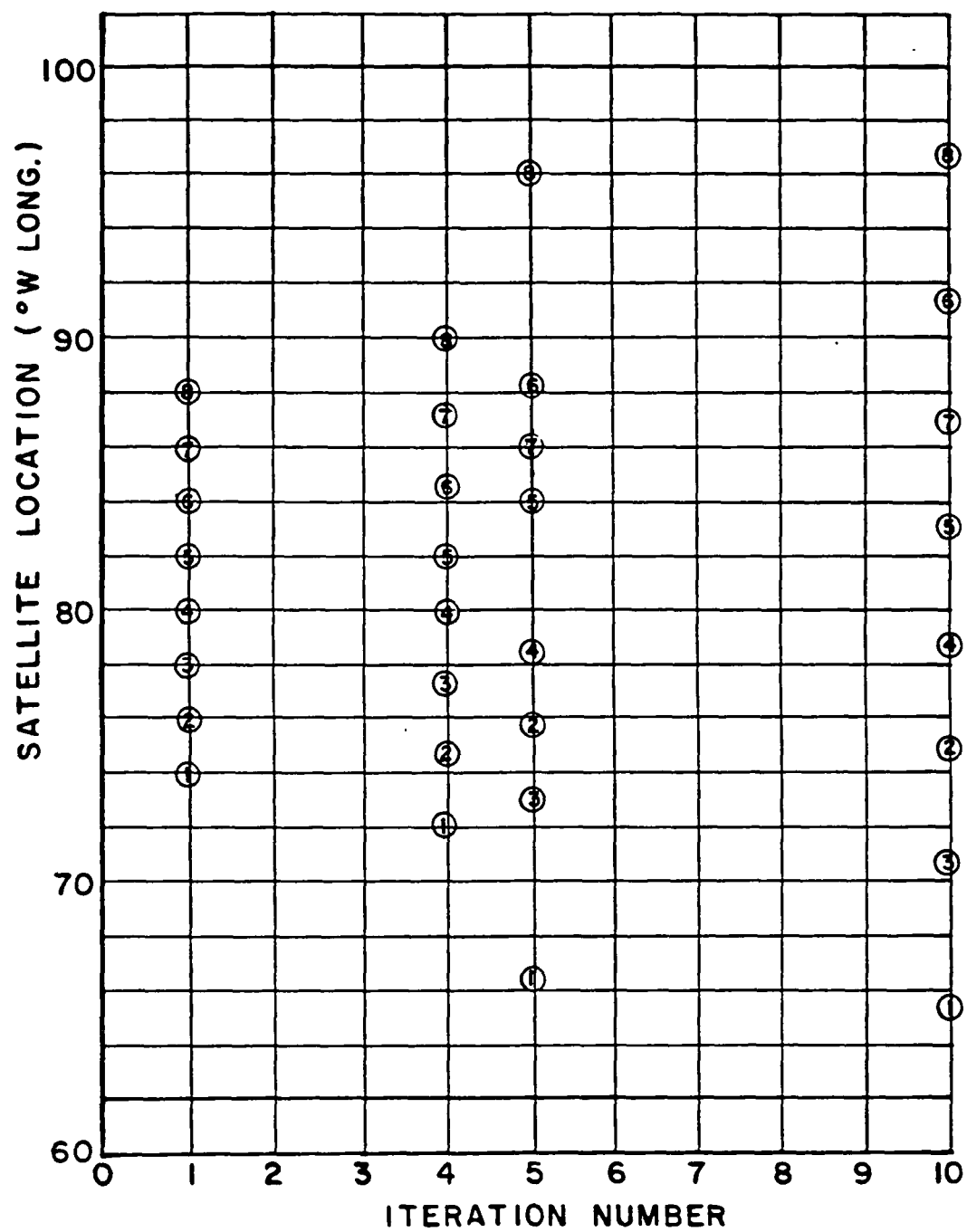


Figure 3. Satellite locations for Run 2 by iteration.

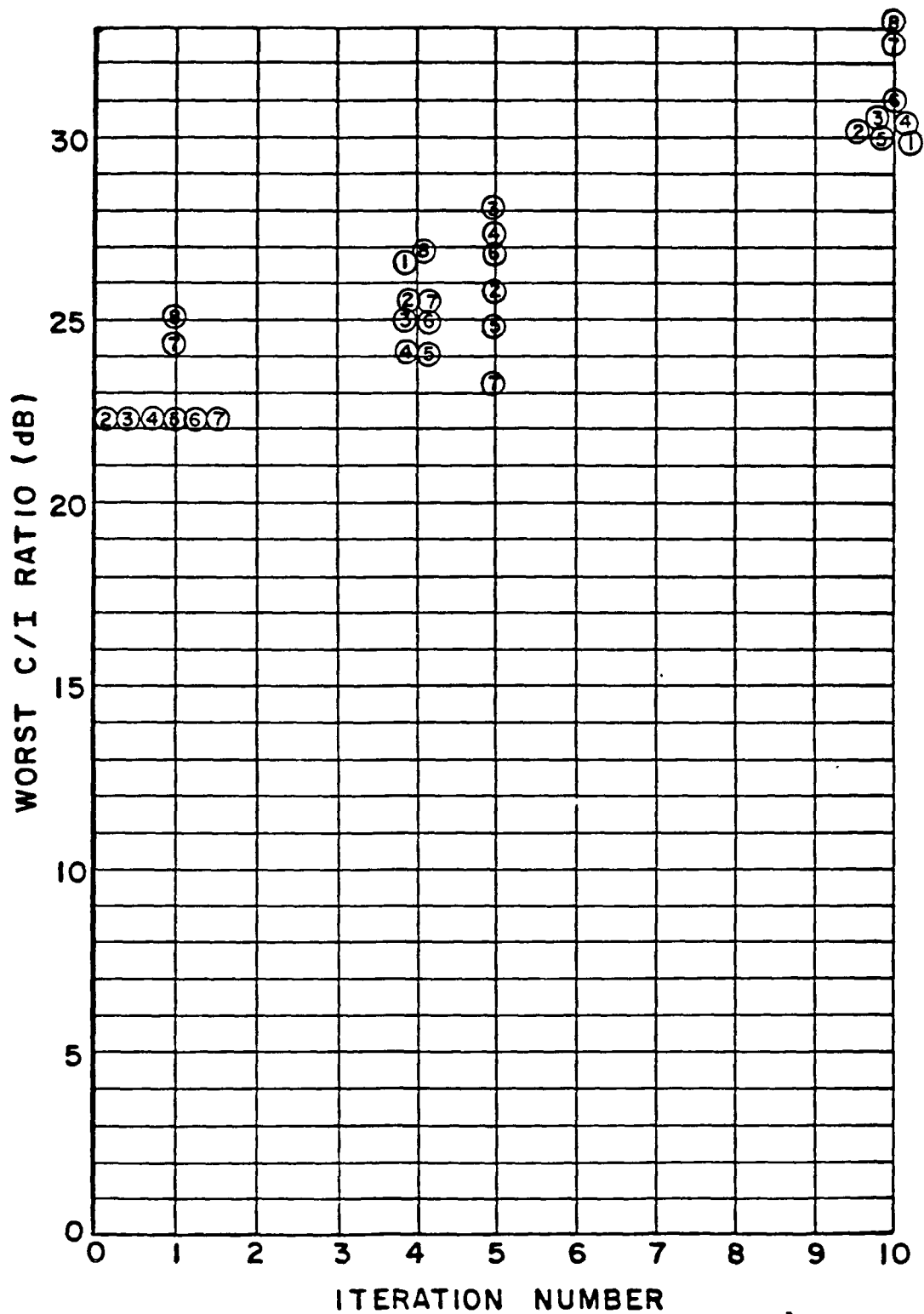


Figure 4. Worst C/I ratios for Run 2 by iteration.

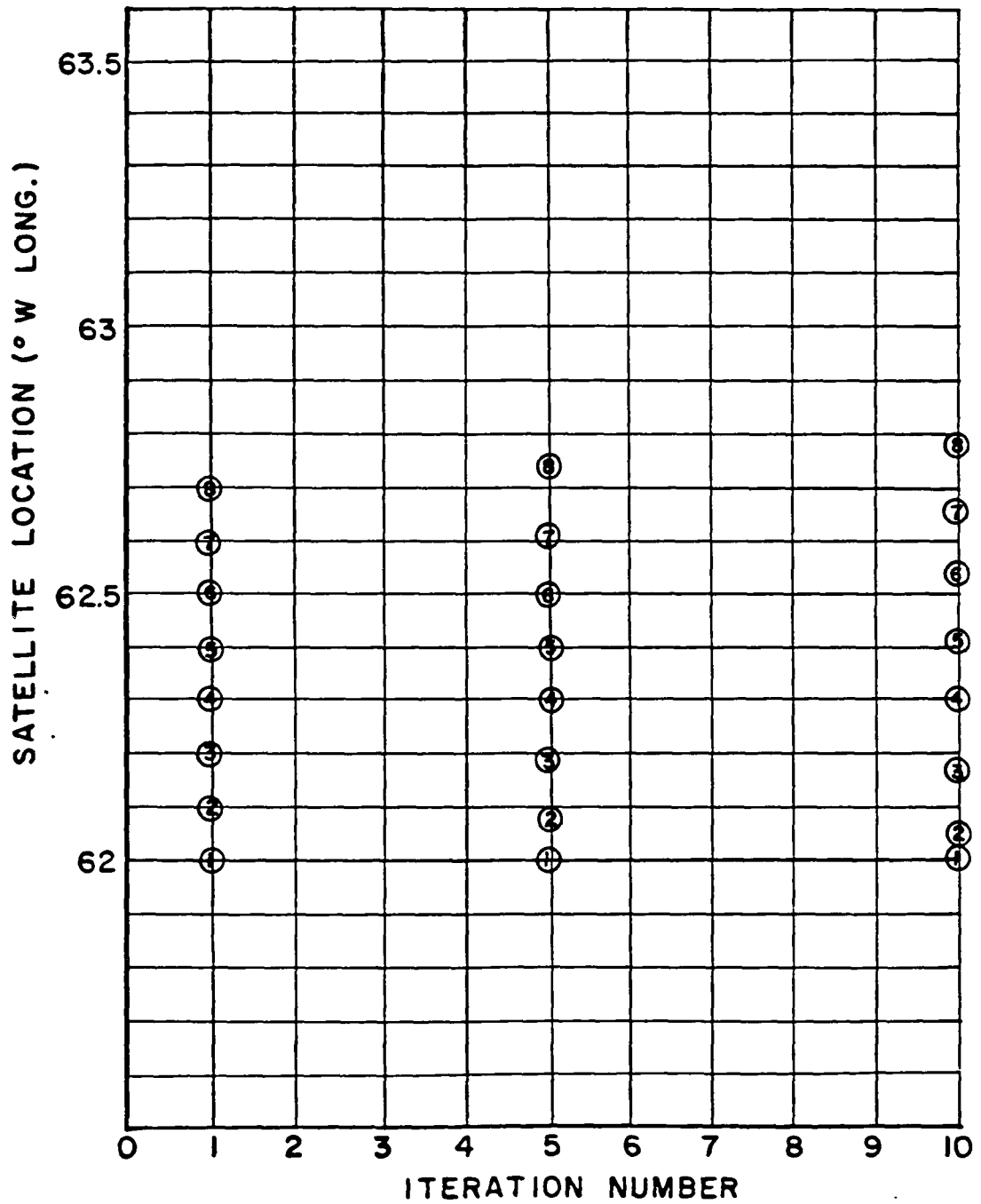


Figure 5. Satellite locations for Run 3 by iteration.

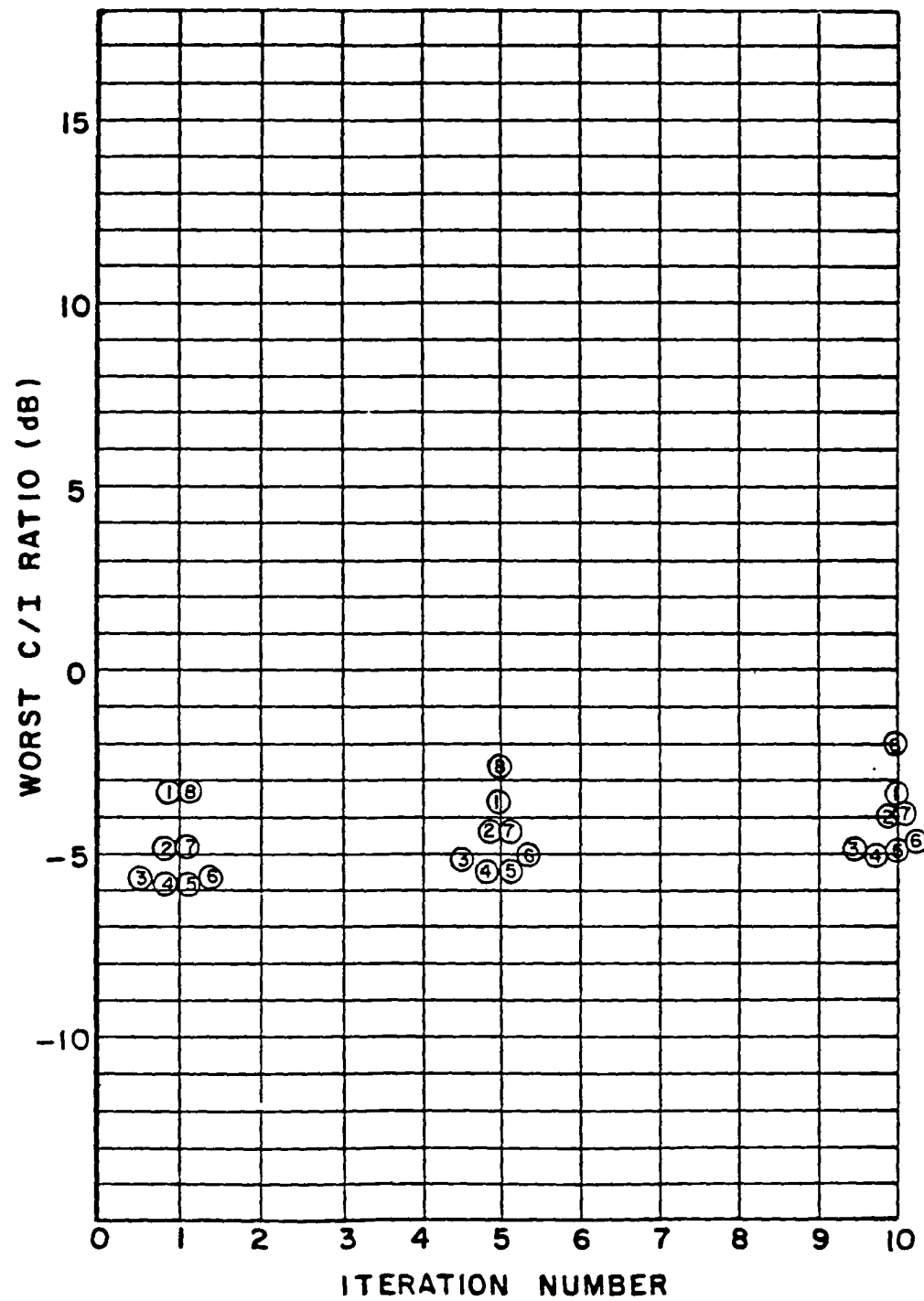


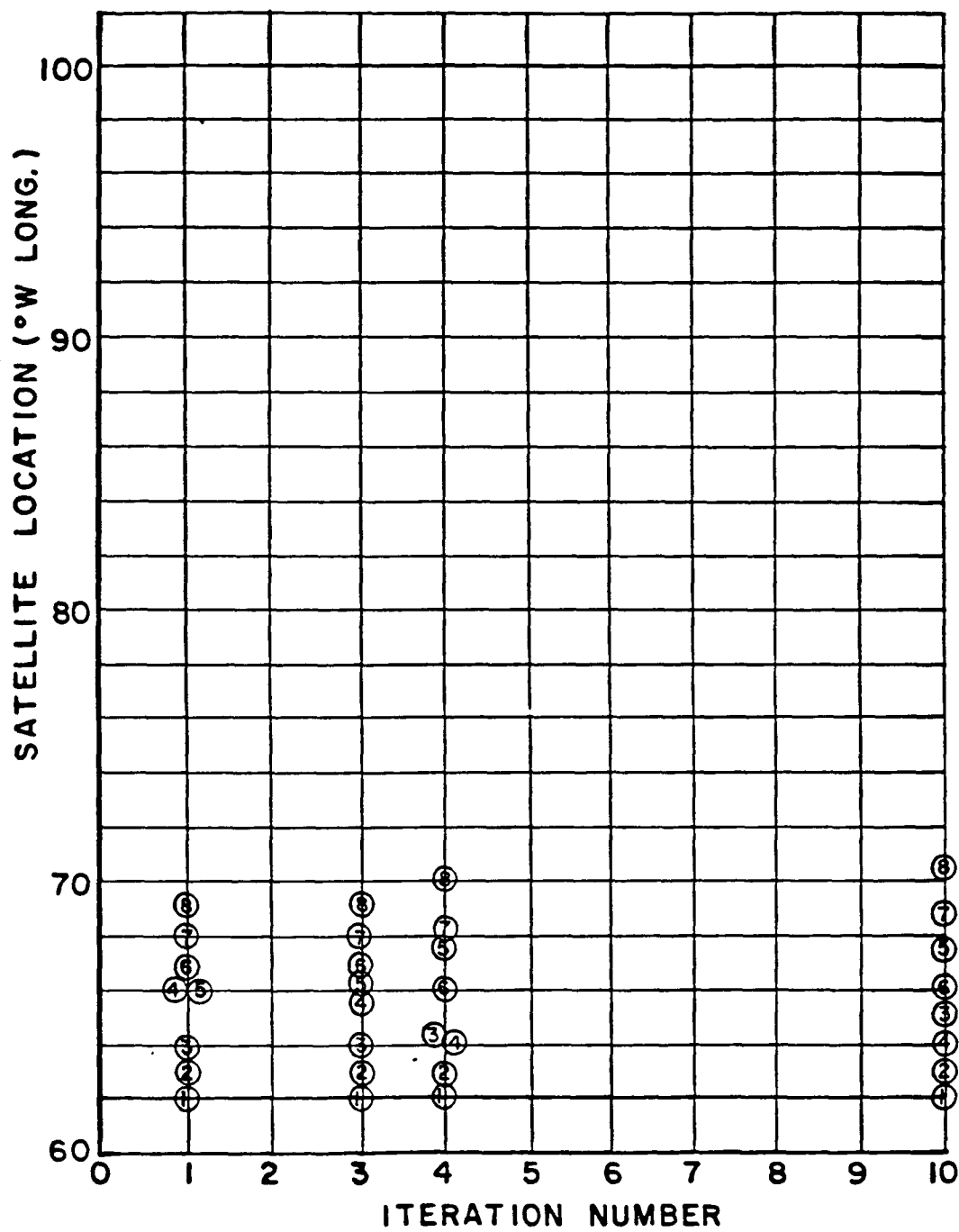
Figure 6. Worst C/I ratios for Run 3 by iteration.



If we recall our previous 10-iteration experiments with all satellites jammed against a boundary, we would expect that convergence would be rather slow for this run. After 10 iterations, this is precisely what we find -- little satellite movement and little improvement in the worst C/I ratios. However, at iterations 15, 19, and 28 we see significant changes in the positions of the satellites and corresponding improvements in the worst C/I ratios. The final solution is actually the best solution found to date. These results indicate that by fixing the length of a run in advance we may terminate the algorithm just before there is a significant improvement in the quality of the solution. This is true even when the initial conditions seem rather unfavorable.

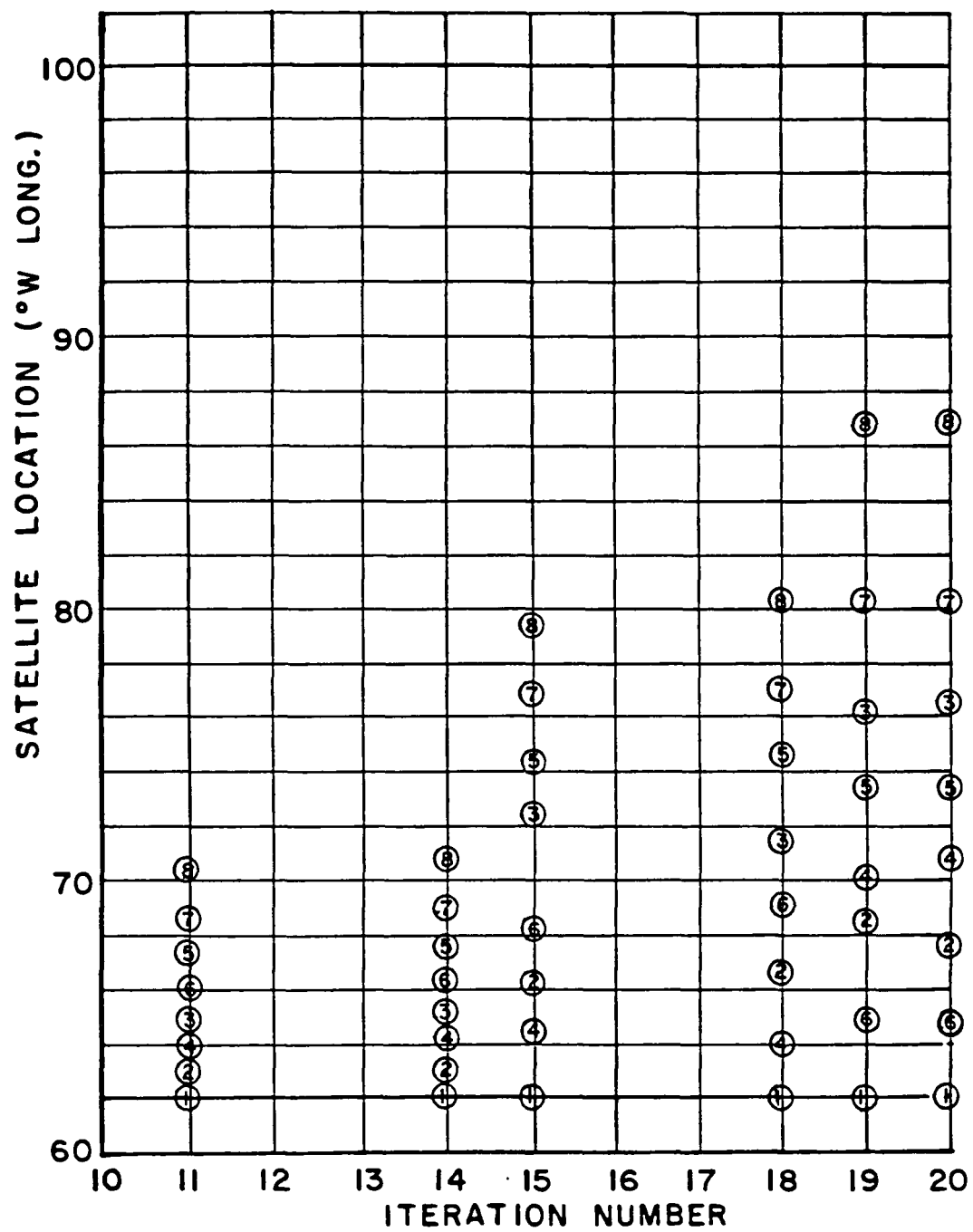
### C. CONCLUSIONS

The conclusions which can be drawn from these experiments with an FSS test problem are quite consistent with those obtained previously with our BSS test problem. It seems that there is an advantage to nearly collocating the satellites near the center of the feasible arc in the initial scenario, at least for shorter runs (10 iterations). The rate of convergence to a good solution is drastically slowed when the satellites are initially located near a boundary. In some of our additional experiments, which are not reported in detail here, we found that by reducing the length of the feasible orbital arc much less attractive solutions than those from Runs 1, 2, and 4 were found. Slow convergence was again evident when the satellites were initially positioned near a boundary.



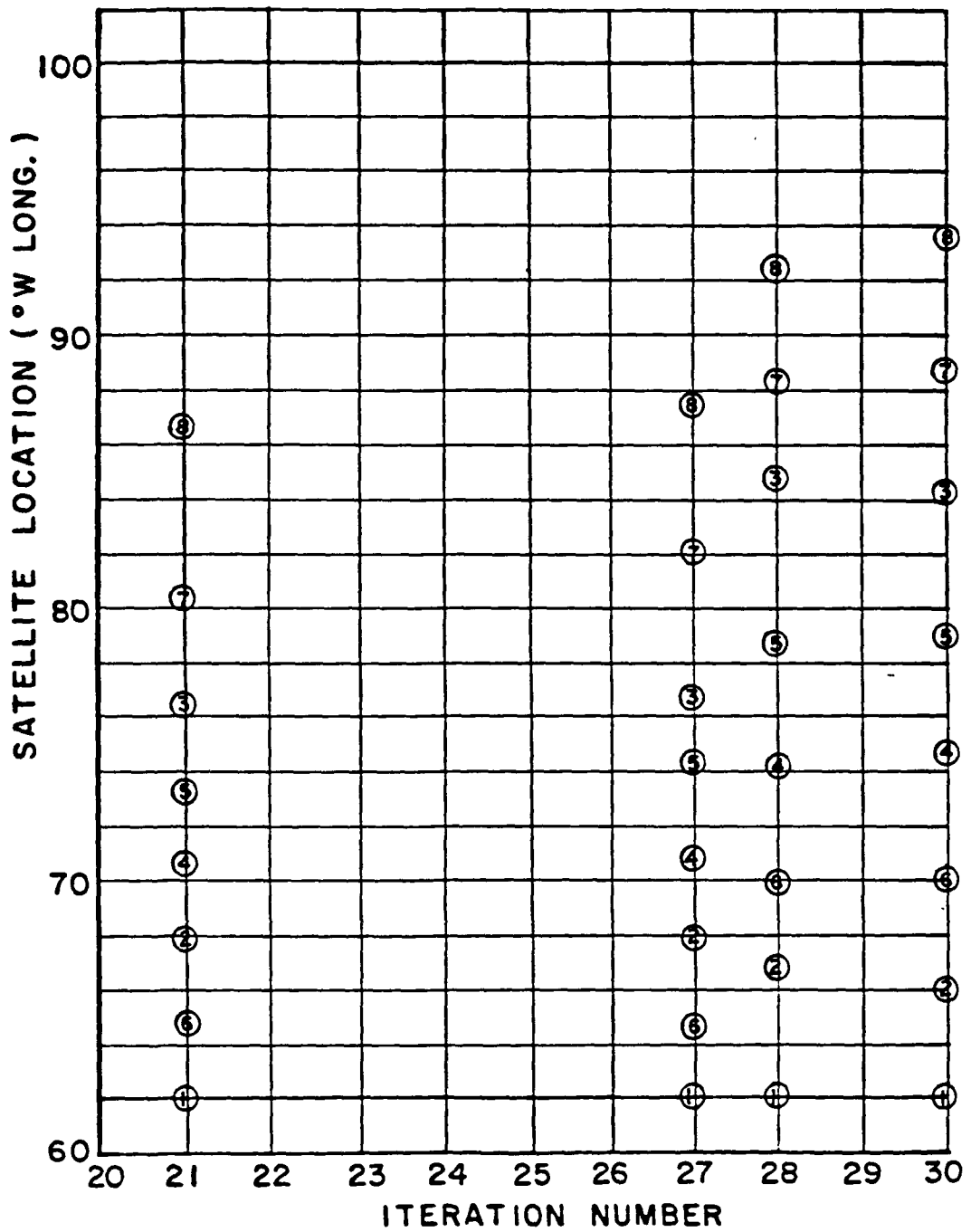
(a) Iterations 1 to 10

Figure 7. Satellite locations for Run 4 by iteration.



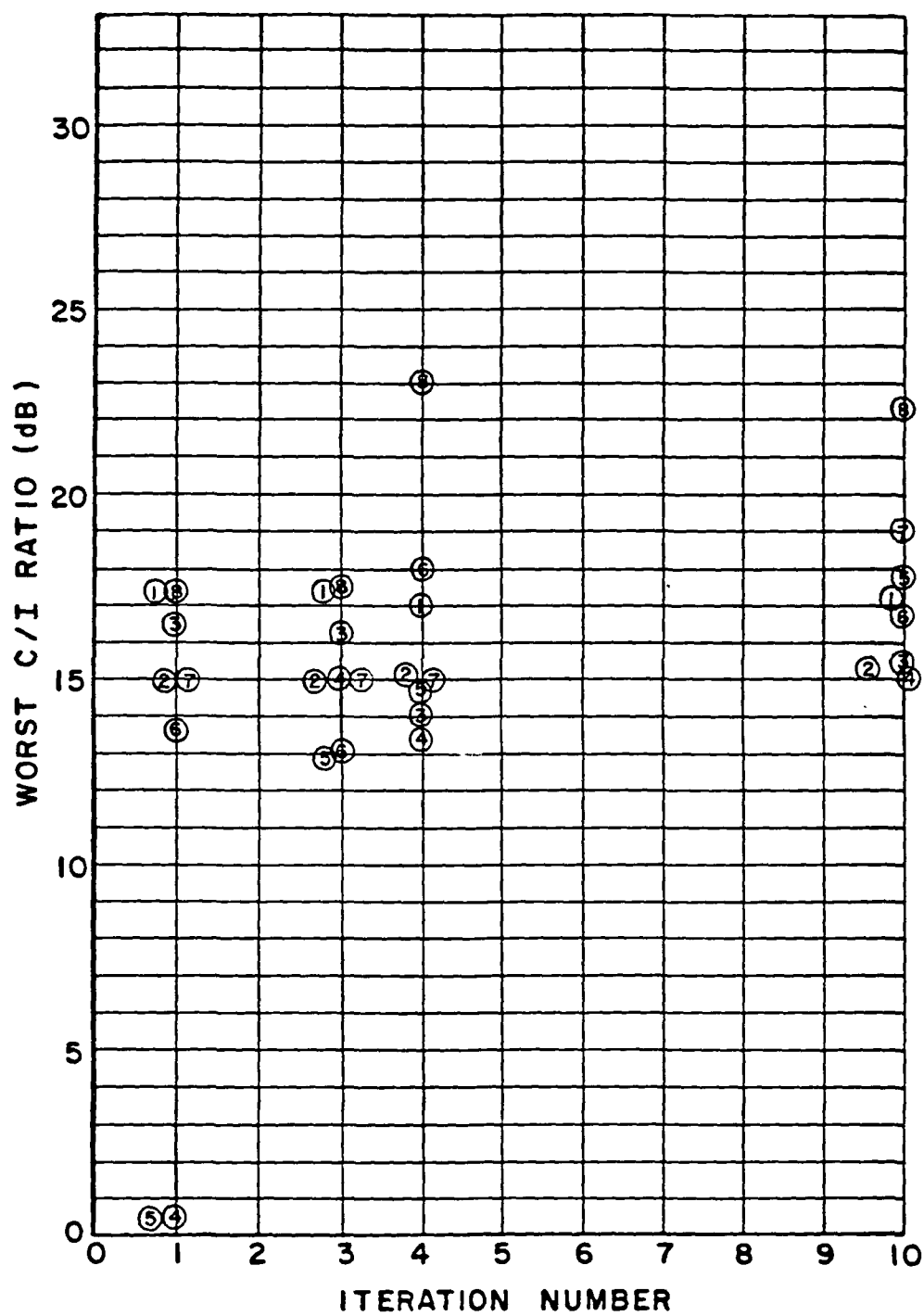
(b) Iterations 11 to 20

Figure 7. (Continued).



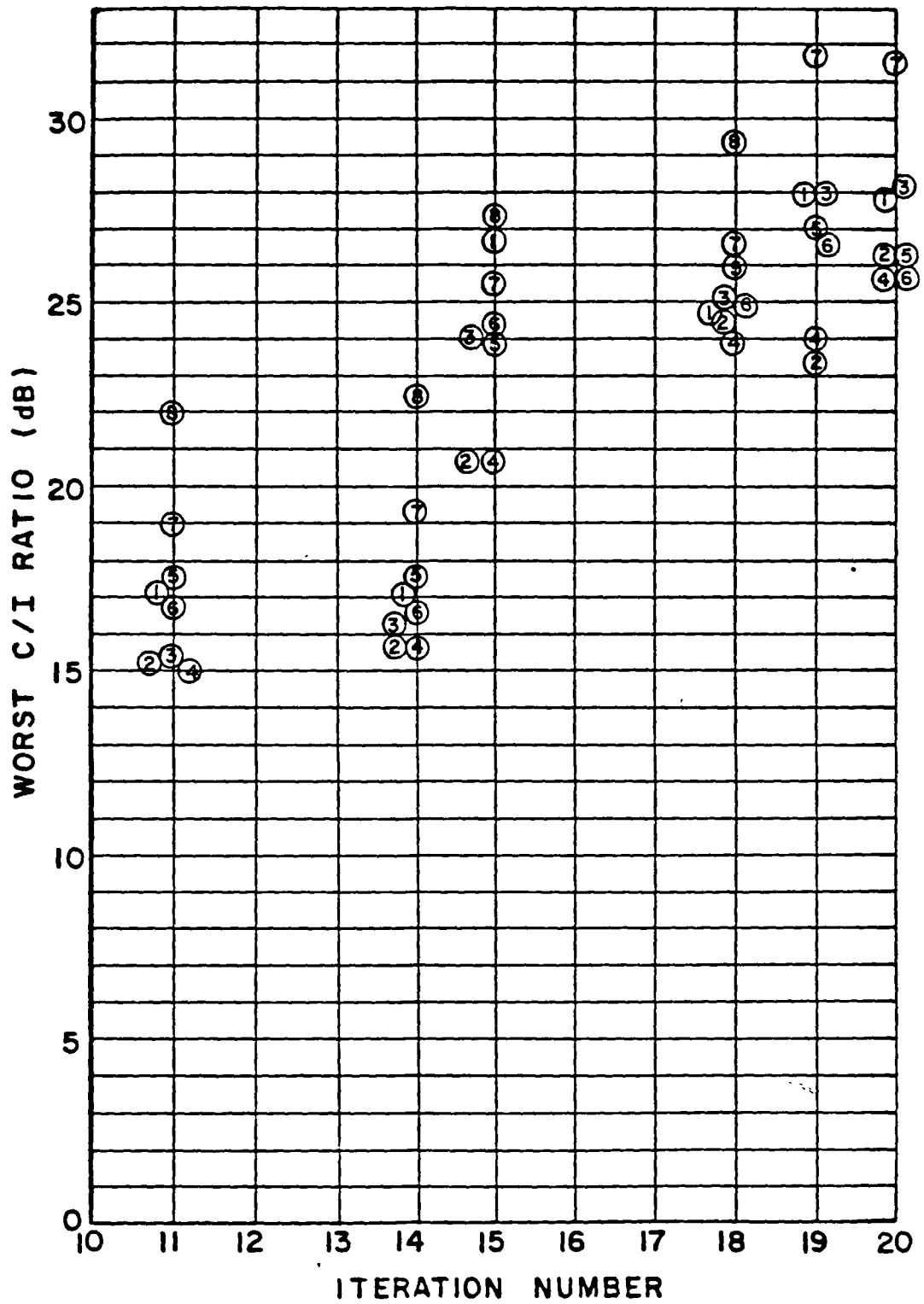
(c) Iterations 21 to 30

Figure 7. (continued).



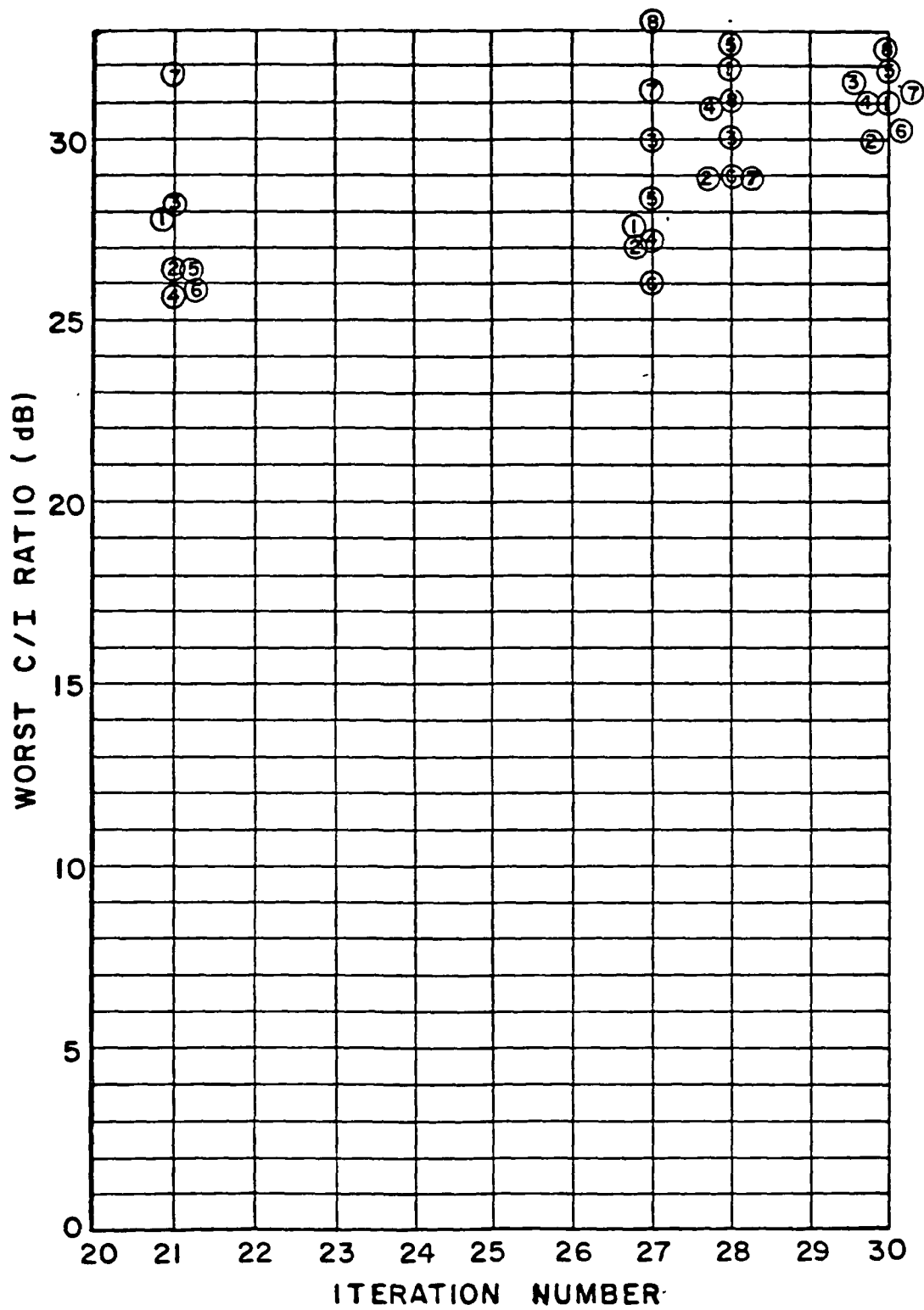
(a) Iterations 1 to 10

Figure 8. Worst C/I ratios for Run 4 by iteration.



(b) Iterations 11 to 20

Figure 8. (Continued).



(c) Iterations 21 to 30

Figure 8. (continued).

#### IV. THE OBJECTIVE FUNCTION

The likelihood of a given degree of success with the gradient-search method depends ultimately on the topography of the objective function hyper-surface; i.e., the number of relative minima and their relative locations and depths in the space defined by the independent variables, the assigned locations and frequencies. In the present case the objective function is so complicated that it is difficult to make general assertions about its topography. It is clear that it is not convex, with "ridges" occurring, at least potentially, when satellites and frequencies are collocated. Each such collocation can be viewed as the boundary between different orderings in orbit/frequency space. The problem can then perhaps be broken into two parts: one related to the ordering, and the second related to the topography for a given ordering. The last is the only one of importance if ordering is of no consequence, e.g., when several satellites serve an identical service area with each using the entire band of available channels, so that reordering is equivalent to renumbering the satellites.

The objective-function topography related to satellite ordering can be visualized by referring to Figure 9, which depicts the satellite coordinates  $S_1, S_2, S_3$  for a 3-satellite system as orthogonal coordinates. The line AH corresponds to  $S_1 = S_2 = S_3$ , i.e., complete collocation; a very high objective-function ridge would be associated with this line of locations. The planes ACHF, ARHE, ADHG ( $S_1 = S_2, S_2 = S_3, S_3 = S_1$ , respectively) correspond to potential ridges of pair-wise



satellite collocations. The height of the corresponding objective-function ridges depends on the separation of the corresponding service areas relative to the earth-station antenna beam widths; this will be discussed in more detail in section V.B. and a future technical report on this Grant [3]. These planes divide the  $S_1, S_2, S_3$  space into six regions, each corresponding to a given ordering. For example, the sextant with vertices HABC corresponds to  $S_1 > S_2 > S_3$ , HADC to  $S_2 > S_1 > S_3$ , and HABG to  $S_1 > S_3 > S_2$ .

The topography within each sextant depends much more strongly on the details of the objective function. In principle, the minima should be easy to find: they correspond to points where the gradient vanishes. The difficulty is that the objective function is a very complicated expression involving piecewise continuous functions, viz., the two antenna discriminations and the relative protection ratio. In their respective regions of interest, the satellite and Earth antenna discriminations are each specified by four continuous segments, and the relative protection ratio is specified by five, so that the derivatives of 80 complicated function combinations with respect to many variables would be involved in a brute-force approach.

An approach that attempts to avoid this difficulty is based on the belief that the topography depends on the general nature of these functions, and especially on their quasi-monotonic behavior for fixed ordering, but not on their detailed nature, so that they may, hopefully, be replaced with simpler functions in determining the general topography of the surface, though not in finding the precise locations of the minima. An approach is to replace the difficult functions with

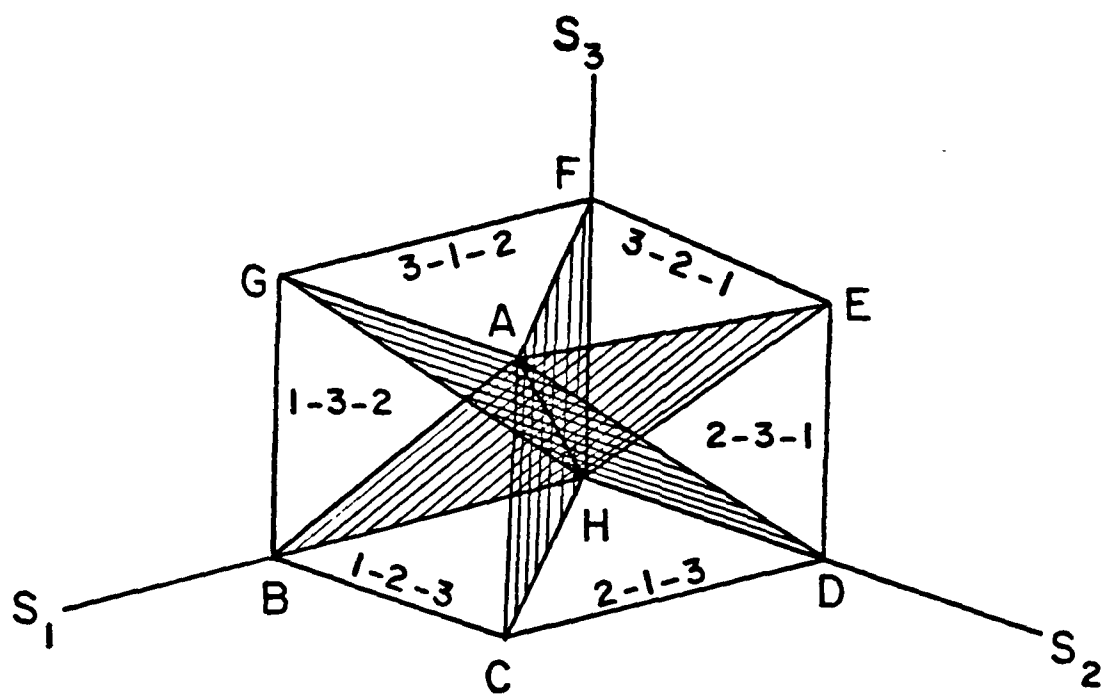


Figure 9. Three-satellite coordinate system for objective-function surface discussion.

"monotonic" transformations. For  $\vec{x} = (x_1, x_2, \dots, x_n)$ , a transformation  $g(\vec{x})$  of the function  $f(\vec{x})$  is defined as monotonic if, for every coordinate pair  $\vec{x}_1, \vec{x}_2$ ,

$$f(\vec{x}_1) < f(\vec{x}_2) \rightarrow g(\vec{x}_1) < g(\vec{x}_2) \quad (1)$$

and

$$f(\vec{x}_1) = f(\vec{x}_2) \rightarrow g(\vec{x}_1) = g(\vec{x}_2) . \quad (2)$$

This line of investigation is only beginning; we have only surmised and not yet proven that such transformations leave the topography unchanged and can therefore be used to explore the properties of minima for fixed ordering.

Such attempts to understand the properties of the objective function will continue during the next interim.

## V. NEW FORMULATIONS FOR FSS SYNTHESIS

### A. INTRODUCTION

Both the gradient search and the cyclic coordinate search algorithms, which we investigated at first for BSS synthesis problems, are applicable to FSS synthesis as well. During the past year an additional class of algorithms has been explored. These algorithms are based on two observations. First, a given desired level of aggregate interference can usually be achieved by specifying a somewhat more stringent single-entry interference level. For example, specifying 35 dB maximum single-entry protection ratios is likely to achieve a 30 dB

aggregate protection ratio\*. Second, the satisfaction of a given single-entry interference protection criterion, such as C/I ratio, can be ensured by sufficient satellite spacing. As discussed below, the required spacing is a function of system parameters, including the locations of the service areas of the two interfering networks. By means of these two observations, the requirement for a given C/I protection ratio is transformed into a set of constraints on the orbital locations of the satellites.

This transformation enables a new set of techniques to be used to attack the problem: the techniques of linear programming and of mixed integer programming. In this domain, the orbital locations are the fundamental variables, and the portion of the space defined by these variables in which all the constraints are met constitutes the feasible region. The minimum requirement then is to find a point in the feasible region. In addition, these programming techniques allow a linear objective function of the orbital location variables to be optimized. Two such functions which have occurred to us are the total occupied orbital arc and the sum of the absolute values of the deviations of the orbital locations from some specified preferred set of locations. Other objective functions may come to mind later.

In each case the first step is the calculation of the constraints. Since these apply to the orbital separations, i.e., differences of the orbital location variables, we refer to this approach informally as the " $\Delta s$  approach" and to the matrix specifying the minimum separations as the " $\Delta s$  matrix".

---

\* The WARC-BS-77 Plan for Regions 1 and 3 used this concept [4].

Two technical reports have been written on the  $\Delta s$  approach and a third is in progress [5,6,7]. Therefore, only a summary of the results is presented here.

## B. REQUIRED SATELLITE SEPARATIONS

Consider the single-entry interference between two down-link satellite communications circuits. The up-link calculation has been shown to be a dual, i.e., of precisely the same form as the down link [7]. The geometry is shown in Figure 10. The following notation is used: S - satellite, E - earth station, W - wanted network, I - interfering network, T - transmit, R - receive. These symbols will also be used as subscripts in the equations below. It should be noted that the angle  $\vec{\psi}_1$  is a two-dimensional vector since, for elliptical or shaped beams, not only its magnitude is important, but also the orientation of its plane with respect to the plane defined by the beam axis and the beam-maximum (or other reference) direction, e.g., the ellipse major axis for elliptical patterns. Similarly the angle  $\vec{\psi}_2$  is a vector, but  $\psi_3$  can be treated as a scalar since there is no incentive for earth stations to use non-circular beams.

The carrier and interference powers can be determined by means of the Friis transmission formula [8] and combined to give a well-approximated single-entry carrier-to-interference ratio [7]

$$(C/I)_{EWR} = \frac{E_{SWT}}{E_{SIT}} \frac{D_{SWT}(\hat{\psi}_1, G_{SWT})}{D_{SIT}(\hat{\psi}_2, G_{SIT}) D_{EWR}(\psi_3, G_{EWR})} \quad , \quad (3)$$

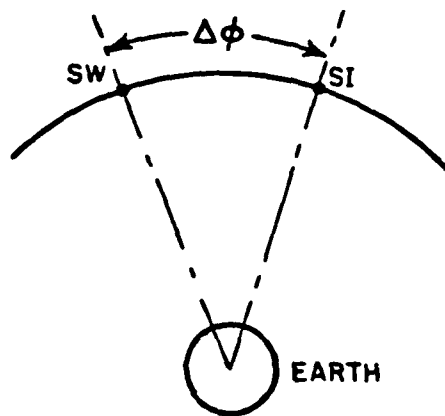
where  $E$  denotes effective isotropic radiated power,  $D$  antenna discrimination relative to the beam maximum, and  $G$  antenna gain in the beam-maximum direction. For satisfactory performance the carrier-to-interference ratio must equal or exceed the required protection ratio, which is the product of a co-channel protection ratio  $P$  and a relative protection ratio  $p(f)$ , where  $f$  denotes the frequency offset from co-channel [9]. Therefore Equation (3) shows that the minimum allowable satellite spacing is implied in

$$R_{DN}^{-1} = D_{SIT}(\hat{\psi}_2, G_{SIT}) D_{EWR}(\psi_3, G_{EWR}) \quad , \quad (4)$$

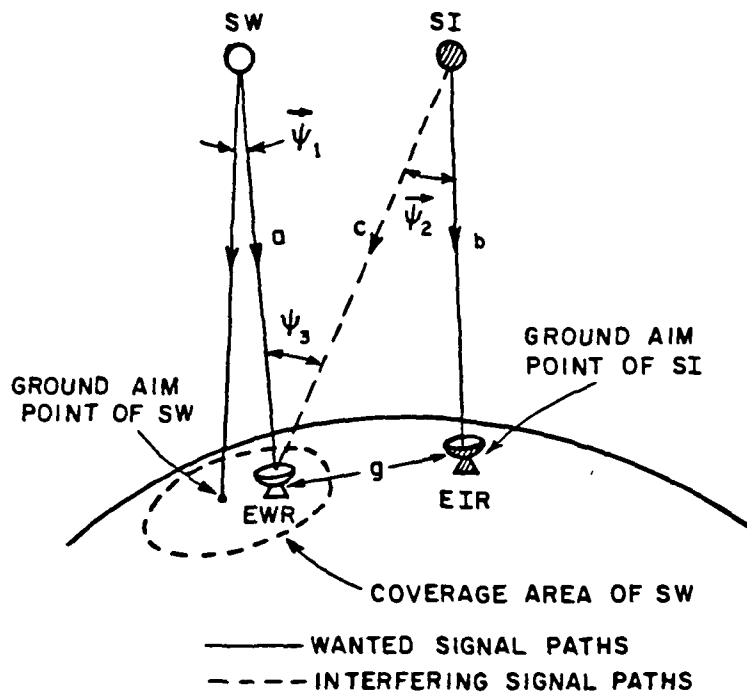
where

$$R_{DN} = P p(f) E_{SWT}^{-1} E_{SIT} D_{SWT}^{-1}(\hat{\psi}_1, G_{SWT}) \quad . \quad (5)$$

The first four factors in  $R_{DN}$  are known system parameters. Also, since calculations will always be performed at test points on the boundary of a service area and since, in practice, satellite beams will be shaped to give a reduction of approximately 3 dB at these test points, one can set  $D_{SWT}(\hat{\psi}_1, G_{SWT}) \approx 1/2$ . The left side of Equation (4) can therefore be considered a known quantity in an orbit synthesis procedure.



(a) Overall geometry



(b) Detail with Earth radius exaggerated for clarity

Figure 10. Interference geometry between down-link networks.

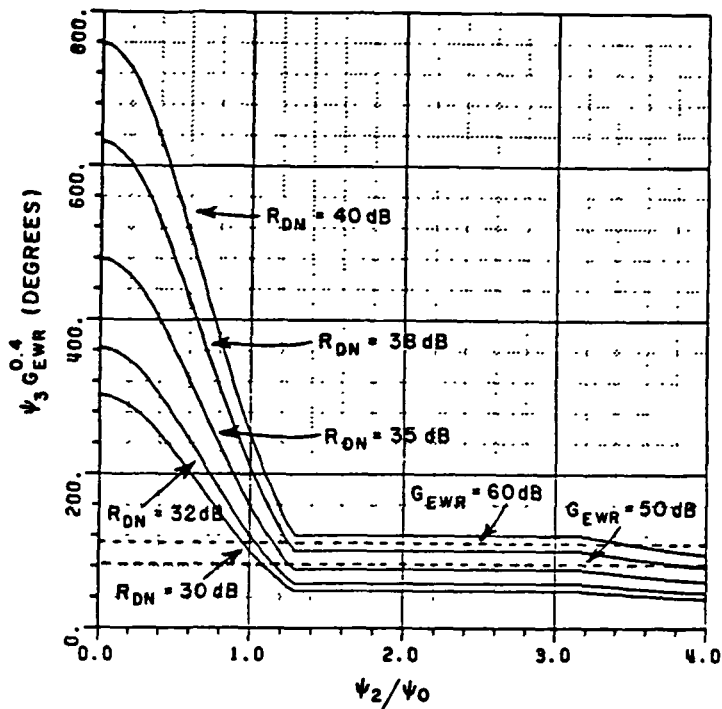
It is important to note that Equation (4) is an implicit equation relating quantitatively the required satellite separation to the separation of the two service areas. The existence of such a relationship has long been recognized qualitatively [10].

### C. SEPARATIONS FOR CIRCULAR BEAMS

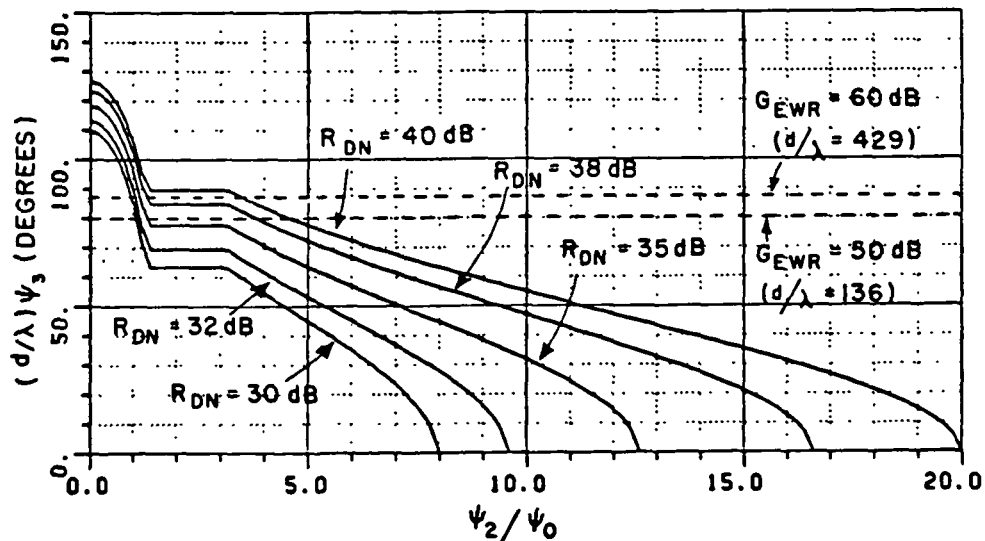
For circular beams, the angle  $\psi_2$  in Equation (4) becomes scalar and it is possible to solve explicitly for  $\psi_3$  as a function of  $\psi_2$  when the discrimination patterns  $D_{SIT}$ ,  $D_{EWR}$  are specified. The relationship can be plotted conveniently as a universal set of contour curves, with  $R_{DN}$  as parameter and normalized values of  $\psi_2, \psi_3$  as coordinates. The universal curves are shown in Figure 11 for discrimination pattern envelopes recommended by the CCIR for co-polarized FSS antennas [11,12]. Unfortunately, no corresponding cross-polarized patterns have been recommended as yet. Two sets of curves are required because of the piecewise CCIR specifications of  $D_{EWR}(\psi_3)$ .

The expression of the universal curves in terms of the antenna-centered "off-axis" angles  $\psi_2$  and  $\psi_3$  is natural and also useful; nevertheless, for system calculations by the  $\Delta s$  approach it is more useful to use the geocentric satellite separation  $\Delta\phi$ , instead of the topocentric angle  $\psi_3$ , as a function of the longitude differences and latitudes of EWR, EIR, and SI, even though this does not allow such a compact, universal presentation. A typical variation of the required separation  $\Delta\phi$  for various system parameters and configurations in terms of longitude and latitude is shown in Figure 12. From these and more





(a) Use for  $\psi_3 > 26.3 G_{EWR}^{-0.3}$  degrees or above appropriate Earth station antenna gain line. (Use numerical  $G_{EWR}$  value unless dB are specified.)



(b) Use for  $\psi_3 < 20 (d/\lambda)^{-1} [5.35 + 5 \log_{10} (d/\lambda)]^{1/2}$  or below appropriate Earth station antenna gain line

Figure 11. Universal curves for the minimum allowable satellite spacing angle  $\psi_3$  as function of the normalized off-axis angle  $\psi_2$ .  $\psi_0$  is the half-power beam width of the satellite antenna,  $d/\lambda$  the diameter-to-wavelength ratio of the EWR antenna.

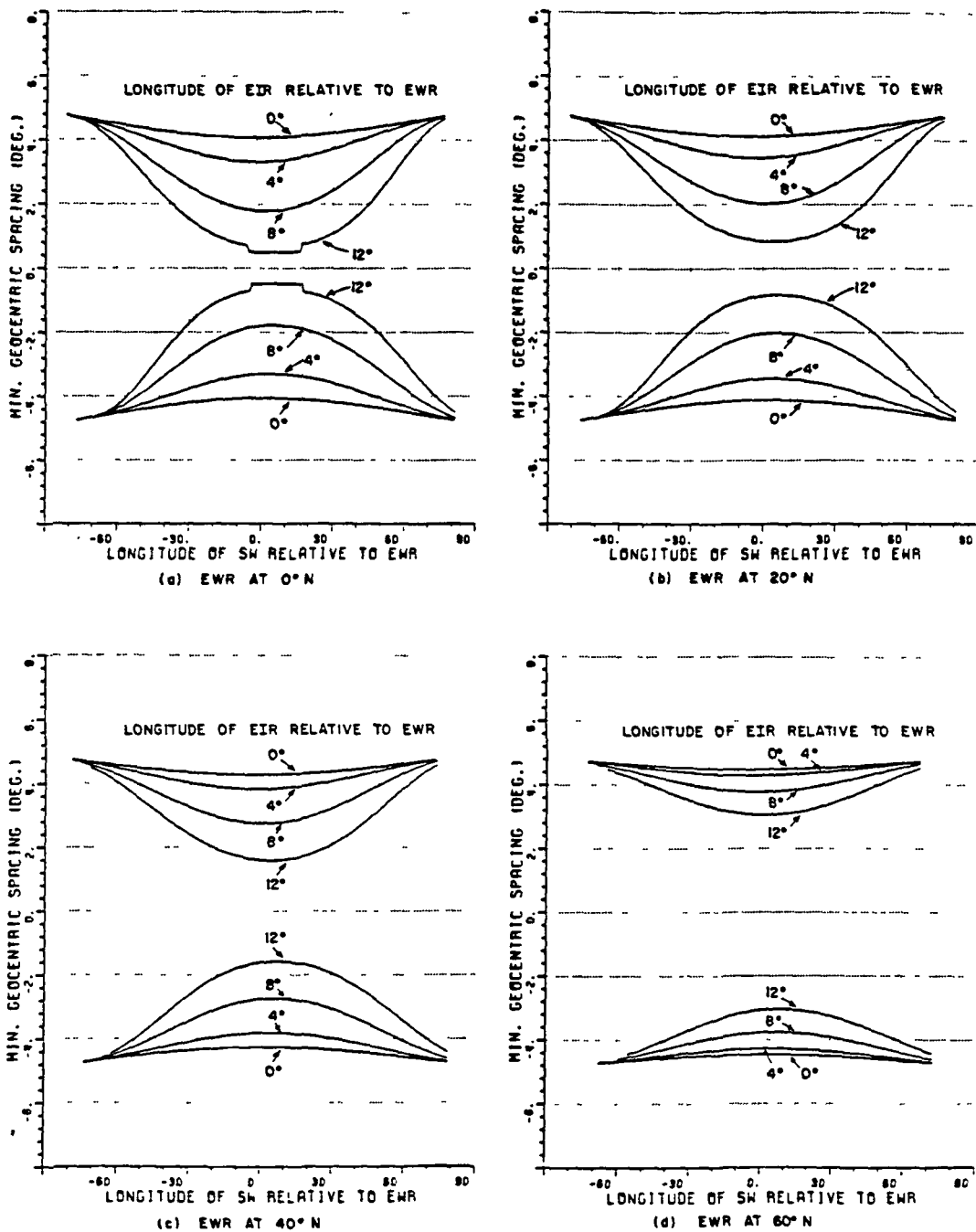


Figure 12. Minimum geocentric satellite spacing when earth stations are separated in longitudinal direction.  $R_{DN} = 35$  dB,  $G_{SIT} = 40$  dB,  $G_{EWR} = 50$  dB.

such computations [13] the following results emerge:

- (a) for practical geometries the smallest required separation occurs when the wanted satellite is near the longitude of the center of its service area,
- (b) for a substantial range of orbital locations about this longitude the required separation varies little.

This last result, which appears to be true also for elliptical beams (see below), is very important in the synthesis procedure because it reduces or eliminates the need to recalculate the required satellite separations as satellite orbit assignments are changed.

#### D. SEPARATIONS FOR ELLIPTICAL BEAMS

For elliptical beams, the required satellite separations can be calculated to a sufficient degree of approximation by a numerical procedure [5,6].

As a demonstration, calculations were performed for a test problem consisting of the six service areas shown in Figure 13; the results are shown in Table 1.

TABLE 1

Δs VALUES IN DEGREES

	BOL	CHL	PRG	PRU	URG
ARG	4.17	4.19	4.32	1.41	4.14
BOL		4.57	4.04	4.26	0.94
CHL			2.00	3.94	1.59
PRG				1.10	2.46
PRU					0.37

## E. LINEAR PROGRAMMING FORMULATION

The FSS synthesis problem can now be formulated as a linear program with a set of nonlinear side constraints. The set of satellite locations which satisfy the constraints constitutes the feasible region. A variety of linear functions can be selected to be optimized. Three functions have occurred to us:

- (a) to search only for some point in the feasible region by setting the function to be minimized equal to zero;
- (b) to minimize the occupied orbital arc;
- (c) to minimize the sum of the absolute deviations of the satellite locations from a specified set of locations.

The last objective has been implemented in the form of both linear and mixed-integer programming codes. Only an overview is given here; the reader is referred to the technical report [5] for more detail.

Linear programs are much more readily solvable than nonlinear programs and integer programs. They are most often solved by the simplex method [15]. This technique examines a sequence of basic solutions to the constraints of the linear program. Each solution examined has an objective function value no less favorable than that of the previous solution. The algorithm terminates when it is determined that no improved solution can be found.

The presence of the nonlinear side constraints prevents us from using the simplex method in its most common form. The method can be modified to handle these additional constraints through the use of restricted basis entry [16]. When employing the simplex method with

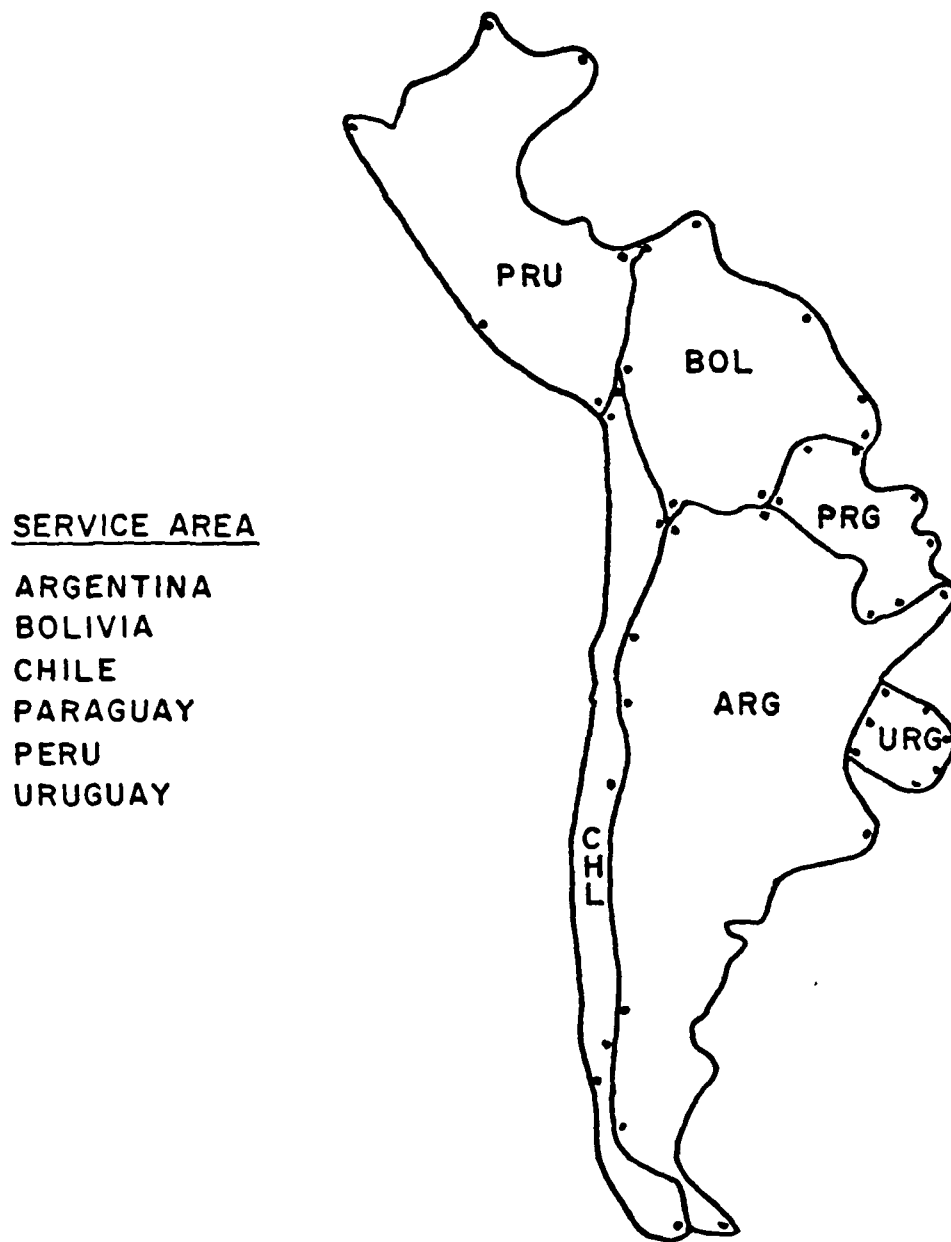


Figure 13. Geography of the six-service-area scenario. Dots indicate test points.

restricted basis entry, we are certain to find a local, but not necessarily a global, optimum. As formulated, the problem has  $m(m+2)$  variables, where  $m$  is the number of satellites, and  $m^2$  constraints, not counting the simple bound and complementarity constraints. The formulation is similar to one suggested by Ignizio for the N-job, single-machine scheduling problem [17].

#### F. MIXED INTEGER PROGRAMMING FORMULATION

As discussed in our technical report [5], the same problem can also be formulated as a mixed integer program [18]. A global optimum is guaranteed when this formulation is employed. However, the computational effort required to find a final solution can be immensely greater than it would be with the linear programming formulation, and this approach may not be suitable for problems involving many satellites. In any case, this formulation is helpful in assessing the quality of the solutions found with the linear programming formulation on small test problems.

If there are  $m$  satellites, the mixed integer formulation entails  $m(m+2)$  continuous variables,  $m(m+1)/2$  binary variables, and  $2m^2-m$  constraints. The time required to solve an FSS synthesis problem with this formulation will be most heavily dependent upon the number of binary variables. For large problems (many satellites), this formulation may involve prohibitive solution times.

## G. NUMERICAL RESULTS

The FSS synthesis minimizing the sum of absolute deviations of orbital positions from a prescribed "desired" set was solved, both as a linear program with the simplex method with restricted basis entry and as a mixed integer program via branch-and-bound [19]. The service areas and test points were those of Figure 13 with one satellite per service area. The available orbital arc for each satellite was specified as  $80^{\circ}\text{W}$  to  $110^{\circ}\text{W}$ . It was assumed that each satellite would carry a full complement of frequency channels, so that a co-channel calculation is appropriate. A single-entry C/I value of 30 dB was chosen with the intent of achieving a 25 dB aggregate co-channel C/I ratio. With these assumptions the  $\Delta s$  values of Table 1 are pertinent. Three problems were run, differing only in the specified "desired" satellite locations. In problem 1, this "desired" location was specified for every satellite as  $95^{\circ}\text{W}$ , the center of the arc. In problem 2, all "desired" locations were specified at  $110^{\circ}\text{W}$ , the westernmost end of the arc. In problem 3, each was specified near the central longitude of the ellipse circumscribing the service area to be served; these "desired" longitudes are indicated in the column labeled DL in Table 2, which shows the solutions obtained for all three problems by both methods. The LP formulations required 48 variables and 36 constraints, while 63 variables, 15 of them binary, and 66 constraints were needed for the MIP formulation.

The solutions to these test problems illustrate some important points. First of all, the solution of a synthesis problem by means of an integer program can require a substantially greater amount of

computer time than by means of a linear program. Secondly, the two approaches used can produce strikingly different solutions. (See the results for Problems 1 and 2.) It may also happen that the same solution will be found with both methods, even though this is not evident from the results presented here. Finally, acceptable solutions, in terms of aggregate co-channel C/I ratios, are obtained even when the objective function value for the linear programming solution differs substantially from that for the mixed integer programming solution, the global optimum. This is not unexpected because the  $\Delta s$  constraints guarantee acceptable single-entry C/I ratios. Table 3 shows the distributions of aggregate co-channel C/I ratios for the two methods and three problems. It will be remembered that a 30 dB single-entry constraint was used to calculate the  $\Delta s$  table on which all these calculations are based.

## VI. CONCLUSIONS

It has proven difficult to test our extended-gradient-search BSS synthesis procedure with the RARC-83 scenario, in part because this scenario contains some ad-hoc "fixes" which have not been described in detail. Recent meetings with NASA/Lewis personnel have shed much light on these matters and have given us a better appreciation and understanding of this scenario. We expect soon to be able to execute the gradient search and cyclic coordinate algorithms using the RARC-83 scenario as an initial solution. Our results are likely to recognize most of the complexities of the international agreement which resulted in this scenario.



TABLE 2  
SOLUTIONS TO TEST PROBLEMS

LOCATIONS (°W)	Problem 1		Problem 2		Problem 3		
	LP	MIP	LP	MIP	DL	LP	MIP
ARG	105.74	88.68	110.00	101.35	87.5	101.26	88.76
BOL	101.57	99.57	104.33	97.18	92.5	92.5	92.93
CHL	97.00	95.00	99.76	105.54	97.5	97.07	97.5
PRG	95.00	93.00	97.76	107.54	87.5	87.5	84.44
PRII	93.06	91.06	108.59	109.63	102.5	102.67	102.50
URG	92.54	96.59	105.86	110.00	82.5	82.5	81.98
objective function (degrees)	23.71	18.42	33.69	28.76		14.36	5.27
orbital arc length (degrees)	13.20	10.89	12.24	12.82		20.17	20.52
CPU time* (sec)	1.31	25.23	1.30	13.39		1.25	2.86

\*IBM-3081 computer

TABLE 3

NUMBER OF TEST POINTS CORRESPONDING TO  
A GIVEN AGGREGATE C/I RATIO RANGE FOR EACH PROBLEM AND METHOD

Problem	Method	C/I Interval, dB				
		<27	27-28	28-30	30-35	>35
1	LP	0	1	8	16	29
1	MIP	0	1	6	18	29
2	LP	0	0	5	20	29
2	MIP	0	4	10	25	15
3	LP	0	0	4	16	34
3	MIP	0	0	9	14	31

We have experimented with a new FSS synthesis test problem, based on a single service area (USA-East, served by 8 satellites) to explore the applicability of gradient search to FSS synthesis. We have shown that the gradient search algorithm is capable of finding good solutions to such a problem. The starting solution and the length of the feasible orbital arc seem to have significant effects on the quality of the final solution. We have also seen by virtue of the results presented herein, that the premature termination of the gradient search algorithm (when the gradient is nonzero) can prevent us from finding a good solution (Run 4). However, there is no way to predict what an adequate number of iterations would be.

The concepts of service areas and minimum satellite separations have proven useful in identifying new formulations and approaches for satellite synthesis problems. By expressing the single-entry protection ratio requirements as a set of satellite-separation requirements, we have been able to formulate the FSS synthesis problem as a linear program with a set of nonlinear side constraints. Even though this set of side constraints does complicate matters somewhat, the computational advantages of linear programming over the gradient and cyclic coordinate algorithms makes this approach appealing. A mixed-integer formulation was also programmed to allow assessment of the linear programming solution quality. The numerical results of three test problems are very encouraging.

## VII. PLANS FOR THE NEXT INTERIM

Our plans for the next interim include:

1. Make gradient and cyclic coordinate search runs with the RARC-83 scenario
2. Conduct a systematic set of experiments with the gradient search and cyclic coordinate algorithms on a small BSS synthesis test problem
3. Continue our study of the significance and applications of the service area and satellite separation concepts
4. Experiment with new formulations of the FSS synthesis problem which take advantage of the service area and satellite separation concepts

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